Functions

Definition:

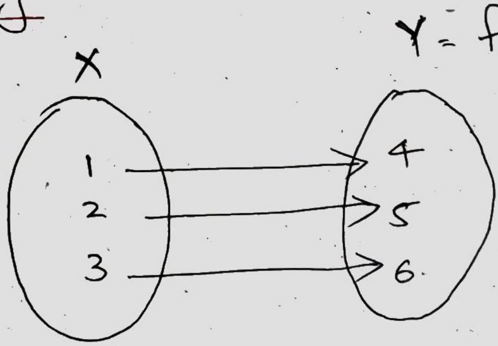
A function or map f from a set X to a set Y is a rule which associates to every element x in X, a unique element in Y, which is denoted by $f(x)$. The element $f(x)$ is called the image of x under f. The function is denoted by $f: X \to Y$

In other words,

A relation f from a set X to a set ~~B~~ Y is said to be a function if every element of set X has one and only one image in set Y.

If f is a function from X to Y and $(x, y) \in f$, then $f(x) = y$, where y is called the image of x under f and x is called the preimage of y under f.

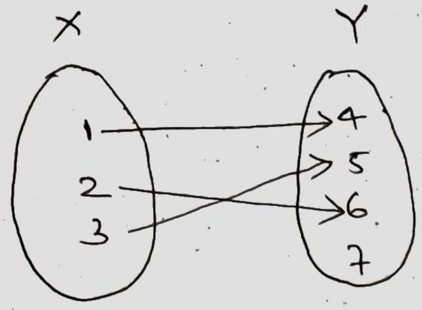
eg: 1



Here, $f(1) = 4$, $f(2) = 5$, $f(3) = 6$

Domain = $\{1, 2, 3\}$, Range = $\{4, 5, 6\}$

eg: 2.



is also a function since every element in X has one and only one image in Y.

Note: The whole set Y is called the Codomain of the function.

Therefore, range $\subset$ Codomain.

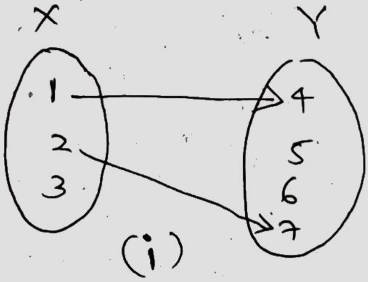
In the above example,

Domain = $\{1, 2, 3\}$

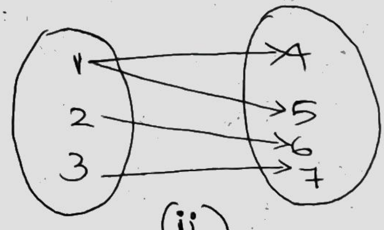
Range = $\{4, 5, 6\}$

Codomain = $\{4, 5, 6, 7\}$

eg: 3



(i)



(ii)

(i) is not a function since element 3 doesnot have an image in Y. (ii) is not a function since '1' has 2 images (4 and 5)

Definition:

$f: X \to Y$ is said to be one-to-one or injective, if different elements in X have different images, i.e, $f(x_1) \neq f(x_2)$ when $x_1 \neq x_2$.

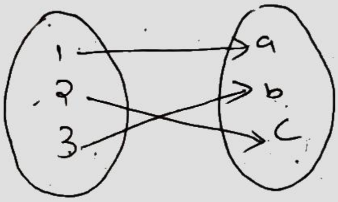
Note: To prove that f is one-to one, we prove the following: Assume $f(x_1) = f(x_2)$ and show that $x_1 = x_2$

Definition:

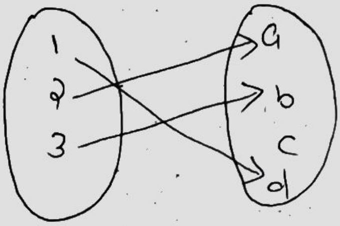
$f: X \to Y$ is onto or surjective if every element y in Y is the image of some element x in X

Definition:

$f: X \to Y$ is said to be one-to-one correspondence or Bijection if f is both one-to-one and onto.



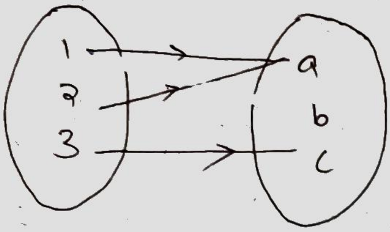
(i)



(ii)

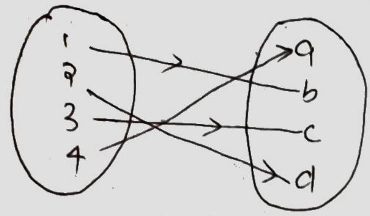
Both (i) and (ii) are one-to-one

However, (ii) is not an onto function



(iii)

Not One-to-one or onto function



(iv)

Bijective function

* $f : Z \to Z$ given by $f(n) = 2n$ is one-to-one but not onto.

Sol: Suppose $f(n_1) = f(n_2)$, Then $2n_1 = 2n_2 \Rightarrow n_1 = n_2$

Hence f is one-to-one It is not onto since no odd integer can be the image of any element in Z

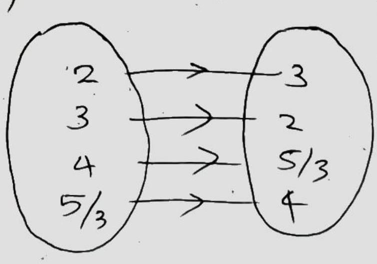
* show that $f : R \to R - \{1\}$ given by $f(x) = \dfrac{x+1}{x-1}$ is onto

Sol: Let $y \in R$. Suppose $y = f(x) = \dfrac{x+1}{x-1}$

Then $y(x-1) = x+1 \Rightarrow yx - x = 1+y$

$\therefore x = \dfrac{1+y}{y-1}$

As $(1+y)/y-1 \in R$ for all $y \neq 1$, y is the image of $(1+y)/(y-1)$ in $R - \{1\}$. Thus, f is onto.



The Pigeonhole Principle[†]

Suppose a postman distributes 51 letters in 50 mailboxes (pigeonholes). Then it is evident that some mailbox will contain at least two letters. This is enunciated as a mathematical principle called the pigeonhole principle.

If n objects are distributed over m places and $n > m$, then some place receives at least two objects.

EXAMPLE 2.14

If we select 11 natural numbers between 1 to 380, show that there exist at least two among these 11 numbers whose difference is at most 38.

Solution

Arrange the numbers 1, 2, 3, . . ., 380 in 10 boxes, the first box containing 1, 2, 3, . . ., 38, the second containing 39, 40, . . ., 76, etc. There are 11 numbers to be selected. Take these numbers from the boxes. By the pigeonhole principle, at least one box will contain two of these eleven numbers. These two numbers differ by 38 or less.