

Results:

$$\bullet \int k f(x) dx = k \int f(x) dx$$

$$\bullet \int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx$$

$$* \int \frac{5}{x} dx$$

$$\text{Sol: } 5 \int \frac{1}{x} dx = \underline{\underline{5 \log x + C}}$$

$$* \int \frac{3}{u} du$$

$$\text{Sol: } 3 \int \frac{1}{u} du = \underline{\underline{3 \log u + C}}$$

$$* \int (\sin x + \cos x) dx$$

$$\text{Sol: } \int \sin x dx + \int \cos x dx$$

$$= \underline{\underline{-\cos x + \sin x + C}}$$

$$* \int (x^3 + 3x^2 + 4) dx$$

$$\text{Sol: } \int x^3 dx + \int 3x^2 dx + \int 4 dx$$

$$= \frac{x^{3+1}}{3+1} + 3 \cdot \left( \frac{x^{2+1}}{2+1} \right) + 4x + C$$

$$= \underline{\underline{\frac{x^4}{4} + x^3 + 4x + C}}$$

$$* \int (1-x) \sqrt{x} \, dx$$

$$\text{Sol: } \int \sqrt{x} \, dx = \int x \cdot \sqrt{x} \, dx$$

$$= \int x^{1/2} \, dx - \int x \cdot x^{1/2} \, dx$$

$$= \int x^{1/2} \, dx - \int x^{3/2} \, dx$$

$$= \frac{x^{1/2+1}}{1/2+1} - \frac{x^{3/2+1}}{3/2+1} + C$$

$$= \frac{x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2} + C$$

$$= \frac{2}{3} \cdot x^{3/2} - \frac{2}{5} \cdot x^{5/2} + C$$

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$$* \int \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 \, dx$$

$$\text{Sol: } = \int \left( (\sqrt{x})^2 + \left( \frac{1}{\sqrt{x}} \right)^2 + 2 \cdot \sqrt{x} \cdot \frac{1}{\sqrt{x}} \right) \, dx$$

$$= \int \left( x + \frac{1}{x} + 2 \right) \, dx$$

$$= \int x \, dx + \int \frac{1}{x} \, dx + \int 2 \, dx$$

$$= \frac{x^2}{2} + \log|x| + 2x + C$$

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$$* \int x^2 \left(1 - \frac{1}{x}\right) dx$$

$$\text{Sol: } \int x^2 dx - \int \frac{x^2}{x} dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} + C$$

$$* \int \sec x (\sec x - \tan x) dx$$

$$\text{Sol: } \int \sec^2 x dx - \int \sec x \tan x dx$$

$$= \tan x - \sec x + C$$

$$* \int e^{\log \sin x} dx$$

$$\text{Sol: } \int \sin x dx = \underline{\underline{-\cos x + C}}$$

$$* \int 3^{x+9} dx$$

$$\text{Sol: } = \int 3 \cdot 3^x dx$$

$$= \int 3^x \cdot 9 dx$$

$$= 9 \int 3^x dx$$

$$= 9 \cdot \frac{3^x}{\log 3} + C$$

$$* \int \frac{x^3 + 5x^2 - 4}{x^2} dx$$

$$\text{Sol: } = \int \left( \frac{x^3}{x^2} + \frac{5x^2}{x^2} - \frac{4}{x^2} \right) dx$$

$$= \int x dx + \int 5 dx - \int \frac{4}{x^2} dx$$

$$= \frac{x^2}{2} + 5x + \frac{4}{x} + C$$

$$* \int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$$

$$\text{Sol: } \int \frac{1/\cos^2 x}{1/\sin^2 x} dx$$

$$= \int \frac{\sin^2 x}{\cos^2 x} dx$$

$$= \int \tan^2 x dx$$

$$= \int (\sec^2 x - 1) dx \quad (1 + \tan^2 x = \sec^2 x)$$

$$= \int \sec^2 x dx - \int 1 dx$$

$$= \underline{\underline{\tan x - x + C}}$$

$$* \int \frac{2 - 3 \sin x}{\cos^2 x} dx$$

$$\text{Sol: } = \int \frac{2}{\cos^2 x} dx - \int \frac{3 \sin x}{\cos^2 x} dx$$

$$= 2 \int \sec^2 x dx - \int 3 \cdot \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} dx$$

$$= 2 \int \sec^2 x dx - 3 \int \tan x \sec x dx$$

$$= \underline{\underline{2 \tan x - 3 \sec x + C}}$$

## Some Standard Results

$$* \int e^{ax} dx = \frac{e^{ax}}{a} + C.$$

$$* \int \sin(ax+b) dx = \frac{-\cos(ax+b)}{a}$$

$$* \int \cos(ax+b) dx = \frac{\sin(ax+b)}{a}$$

$$* \int \tan x dx = \log |\sec x|$$

$$* \int \cot x dx = \log |\sin x|$$

$$* \int \tan(ax+b) dx = \frac{\log |\sec(ax+b)|}{a}$$

$$* \int \cot(ax+b) dx = \frac{\log |\sin(ax+b)|}{a}$$

$$* \int \sec x dx = \log |\sec x + \tan x|$$

$$* \int \sec(ax+b) dx = \frac{\log |\sec(ax+b) + \tan(ax+b)|}{a}$$

$$* \int \operatorname{cosec} x dx = \log |\operatorname{cosec} x - \cot x|$$

$$* \int \operatorname{cosec}(ax+b) dx = \frac{\log |\operatorname{cosec}(ax+b) - \cot(ax+b)|}{a}$$

$$* \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x.$$

$$* \int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x$$

$$* \int \frac{1}{1+x^2} dx = \tan^{-1} x$$

$$* \int \frac{-1}{1+x^2} dx = \cot^{-1} x.$$

$$* \int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x.$$

$$* \int \frac{-1}{x\sqrt{x^2-1}} dx = \operatorname{cosec}^{-1} x.$$

## Integrals of some particular functions

$$* \int \frac{1}{x^2+a^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + C.$$

$$* \int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C.$$

$$* \int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$$

$$* \int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$* \int \frac{1}{\sqrt{x^2+a^2}} dx = \log \left| x + \sqrt{x^2+a^2} \right| + C$$

$$* \int \frac{1}{\sqrt{x^2-a^2}} dx = \log \left| x + \sqrt{x^2-a^2} \right| + C$$

$$* \int \frac{1}{ax+b} dx = \frac{1}{a} \log |ax \pm b| + C.$$

$$* \int (ax+b)^n dx = \frac{1}{a} \frac{(ax+b)^{n+1}}{n+1} + C.$$