Relations

A relation R in a set S is a collection of ordered pairs of elements in S (i.e, a subset of $S \times S$). When (x, y) is in R, we write xRy. When (x, y) is not in R, we write $xR'y$

The set of all x-values is called the Domain and set of all y-values is called the Range.

For example, if $S = \{1, 2, 3 \dots 10\}$. We construct the ordered pairs (x, y) such that xRy where $y = 2x$

Now, $S \times S = \{(1,1), (1,2), (2,1), (2,3), (4,10), (3,6), (5,7), \dots \}$

The Relation R will be a subset of $S \times S$.

$\therefore R = \{(1,2), (2,4), (3,6), (4,8), (5,10)\}$

Here, Domain $= \{1, 2, 3, 4, 5\}$

Range $= \{2, 4, 6, 8, 10\}$

Properties of Relations

1) A relation R in S is <u>reflexive</u> if xRx for every x in S.

2) A relation R in S is <u>Symmetric</u> if for x, y in S, yRx whenever xRy.

3) A relation R in S is <u>transitive</u> if x, y and z in S, xRz whenever xRy and yRz.

eg: A relation R in $\{1, 2, 3, 4\}$ is given by

a) $\{(1,1), (2,2), (3,3), (4,4)\}$ – reflexive

b) $\{(1,2), (2,1)\}$ – symmetric

c) $\{(1,2), (2,3), (1,3)\}$ – Transitive

d) $\{(1,2), (2,1), (2,3)\}$ – Not Symmetric
since $(3,2)$ not present

e) $\{(1,2), (2,1)\}$ – Not transitive
since $(1,1)$ not present

f) $\{(1,1), (2,2), (3,3), (4,4), (2,3), (3,2)\}$
↳ Reflexive, Symmetric and transitive.

* A relation R in {1, 2, 3 ... 10} is defined by aRb if a divides b.

Then R is reflexive and transitive (How?) and not Symmetric (How?)

Note: A relation R in a set S is called an Equivalence relation if it is reflexive, symmetric and transitive.

Closure of Relations

A given relation R may not be reflexive or transitive. By adding more ordered pairs to R we can make it reflexive or transitive.

Definition: Let R be a relation in a set S. Then the transitive closure of R (denoted by R^+) is the smallest transitive relation containing R.

Definition: Let R be a relation in S. Then the reflexive - transitive closure of R (denoted by R^*) is the smallest reflexive and transitive relation containing R.

Note: $R^* = R^+ \cup \{(a, a) \mid a \in S\}$

* Let $R = \{(1,2), (2,3), (2,4)\}$ be a relation in $\{1, 2, 3, 4\}$. Find R^+ and R^*.

Sol: Here, $R = \{(1,2), (2,3), (2,4)\}$.

1 is related to 2 and 2 is related to 3 and 4.

R^+ denotes the minimum ordered pairs that should be added inorder to make the relation transitive.

$\therefore$ (1, 3) and (1, 4) should be added.

$\therefore R^+ = \{(1,2), (2,3), (2,4), (1,3), (1,4)\}$

$R^* = R^+ \cup \{(1,1), (2,2), (3,3), (4,4)\}$

$\therefore R^* = \{(1,2), (2,3), (2,4), (1,3), (1,4), (1,1),$
$(2,2), (3,3), (4,4)\}$

Note: The symmetric closure of relation R in a set S is given by

$R = R \cup \{(b, a) \mid aRb\}$